Math 1411 Quiz 2 Name KEY

There are three (3) problems worth 10 points each. **Due Monday, 9/16. Show all of your work to receive full/partial credit.**

1) Find the constant a such that the function is continuous over the entire real line.

$$f(x) = \begin{cases} 2x^3, & x \leq -1 \\ ax + 4, & x > -1 \end{cases}$$

Need $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (2x^3) = 2(-1)^3 = -2$

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (ax+4) = a(-1) + 4 = -a + 4$

so $-a + 4 = -2 \rightarrow a = 6$

2) Find the vertical asymptotes (if any) of the function.

$$h(x) = \frac{x^2 - 4}{x^3 - 2x^2 - x + 2}$$

$h(x) = \dfrac{(x+2)(x-2)}{x^2(x-2) - (x-2)} = \dfrac{(x+2)(x-2)}{(x-2)(x^2-1)} = \dfrac{(x+2)(x-2)}{(x-2)(x+1)(x-1)}$

since $x-2$ is in both numerator and denominator,

there is a hole at $x = 2$

Vertical asymptotes at $x = 1$, $x = -1$

3) Find the derivative by the limit process. If you don't use the limit definition to find the derivative, you will receive **no credit**!

$$f(x) = 2x^2 - 1$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 - 1 - (2x^2 - 1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x^2 + 4x\Delta x + 2\Delta x^2 - 1 - 2x^2 + 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4x\Delta x + 2\Delta x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(4x + 2\Delta x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (4x + 2\Delta x) = 4x$$